

FUNDAMENTAL GROUPS, SLALOM CURVES AND EXTREMAL LENGTH

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To the memory of my teacher and collaborator Viktor Havin, his enthusiasm and his ability to convey a great feeling of the beauty of mathematics

ABSTRACT. We define the extremal length of elements of the fundamental group of the twice punctured complex plane and give upper and lower bounds for this invariant. The bounds differ by a multiplicative constant. The main motivation comes from 3-braid invariants and their application.

In this paper we will describe a conformal invariant for the elements of the fundamental group $\pi_1(\mathbb{C} \setminus \{-1, 1\}, 0)$ of the twice punctured complex plane with base point 0 and give upper and lower bounds for this invariant. The group $\pi_1(\mathbb{C} \setminus \{-1, 1\}, 0)$ is a free group with two generators. We choose generators a_1 and a_2 so that a_1 is represented by a simple closed curve α_1 with base point 0 which surrounds the point -1 counterclockwise such that the image of the curve except the point 0 is contained in the left half-plane. Respectively, a standard representative α_2 of the generator a_2 surrounds the point 1 counterclockwise and the image of the curve except the point 0 is contained in the right half-plane.

The fundamental group $\pi_1 \stackrel{def}{=} \pi_1(\mathbb{C} \setminus \{-1, 1\}, 0)$ is isomorphic to the relative fundamental group $\pi_1^{tr} \stackrel{def}{=} \pi_1(\mathbb{C} \setminus \{-1, 1\}, (-1, 1))$ whose elements are homotopy classes of curves in $\mathbb{C} \setminus \{-1, 1\}$ with end points on the interval $(-1, 1)$. We refer to π_1^{tr} as fundamental group with totally real boundary values (*tr*-boundary values for short). To establish the isomorphism one has to use that the set $(-1, 1)$ is connected and simply connected and contains 0. In the same way π_1 is isomorphic to the relative fundamental group with perpendicular bisector boundary values $\pi_1^{pb} \stackrel{def}{=} \pi_1(\mathbb{C} \setminus \{-1, 1\}, i\mathbb{R})$ whose elements are homotopy classes of curves in $\mathbb{C} \setminus \{-1, 1\}$ with end points on the imaginary axis $i\mathbb{R}$. For an element $w \in \pi_1$ we denote by w_{tr} , and by w_{pb} , respectively, the elements in π_1^{tr} , and in π_1^{pb} , respectively, corresponding to w .

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Consider a rectangle \mathcal{R} with sides parallel to the axes and with length of the horizontal sides equal to \mathbf{b} and length of the vertical sides equal to \mathbf{a} . Recall that according to Ahlfors's definition [1] the extremal length $\lambda(\mathcal{R})$ of such a rectangle is equal to $\frac{\mathbf{a}}{\mathbf{b}}$ and its conformal module $m(\mathcal{R})$ equals $\frac{\mathbf{b}}{\mathbf{a}}$. A continuous mapping of the rectangle \mathcal{R} into $\mathbb{C} \setminus \{-1, 1\}$ is said to represent w_{tr} if it has a continuous extension to the closure of \mathcal{R} which maps horizontal sides to the interval $(-1, 1)$ and whose restriction to each vertical side represents w_{tr} . We make the respective convention for w_{pb} instead of w_{tr} .

We are now in the position to define the extremal length of elements of the relative fundamental groups.

Definition 1. *For an element w of the fundamental group $\pi_1(\mathbb{C} \setminus \{-1, 1\}, 0)$ the extremal length of w with perpendicular bisector boundary values (pb-boundary values for short) is defined as*

$$\lambda_{pb}(w) \stackrel{\text{def}}{=} \inf \{ \lambda(\mathcal{R}) : \mathcal{R} \text{ admits a holomorphic map to } \mathbb{C} \setminus \{-1, 1\} \text{ that represents } w_{pb} \}.$$

An analogous definition can be given for λ_{tr} .

We will give upper and lower bounds for λ_{pb} and λ_{tr} differing by a multiplicative constant. This is of independent interest for the fundamental group of the twice punctured plane, but the main motivation was to give estimates of conformal invariants of braids. Recall that a pure geometric n -braid with base point is a continuous mapping of the unit interval $[0, 1]$ into n -dimensional configuration space $C_n(\mathbb{C}) = \{(z_1, \dots, z_n) : z_j \neq z_k \text{ for } j \neq k\}$ whose values at the endpoints are equal to a given base point in $C_n(\mathbb{C})$. More geometrically, a pure geometric n -braid consists of n pairwise disjoint curves in the cylinder $[0, 1] \times \mathbb{C}$, each joining a point in the top $\{1\} \times \mathbb{C}$ of the cylinder with its copy in the bottom so that for each curve the canonical projection to the interval $[0, 1]$ is a homeomorphism. A pure n -braid with base point is an isotopy class of pure geometric n -braids with fixed base point.

Consider a pure geometric 3-braid. Associate to it a curve in $\mathbb{C} \setminus \{-1, 1\}$ as follows. For a point $z = (z_1, z_2, z_3) \in C_3(\mathbb{C})$ we denote by M_z the Möbius transformation that maps z_1 to 0, z_3 to 1 and fixes ∞ . Then $M_z(z_2)$ omits 0, 1 and ∞ . Notice that z_2 is equal to the cross ratio $(z_2, z_3; z_1, \infty) = \frac{z_2 - z_1}{z_3 - z_1} \cdot \frac{z_3 - \infty}{z_2 - \infty} = \frac{z_2 - z_1}{z_3 - z_1}$.

Let $\gamma(t) = (\gamma_1(t), \gamma_2(t), \gamma_3(t))$, $t \in [0, 1]$, be a curve in $C_3(\mathbb{C})$. Associate to it the curve $\mathfrak{C}(\gamma)(t) \stackrel{\text{def}}{=} 2 \frac{\gamma_2(t) - \gamma_1(t)}{\gamma_3(t) - \gamma_1(t)} - 1$, $t \in [0, 1]$, in \mathbb{C} which omits the points -1 and 1 . If γ is a loop with base point $\gamma(0) = (-1, 0, 1)$ then $\mathfrak{C}(\gamma)$ is a loop with base point $\mathfrak{C}(\gamma)(0) = 0$. The homotopy class of $\mathfrak{C}(\gamma)$ in $\mathbb{C} \setminus \{-1, 1\}$ with base point 0 depends only on the homotopy class of γ in the configuration space $C_3(\mathbb{C})$ with base point $(-1, 0, 1)$. We obtain a surjective homomorphism \mathfrak{C}_* from

the fundamental group of $C_3(\mathbb{C})$ with base point $(-1, 0, 1)$ to the fundamental group of $\mathbb{C} \setminus \{-1, 1\}$ with base point 0. The kernel of \mathfrak{C}_* equals $\langle \Delta_3^2 \rangle$, the subgroup of \mathcal{B}_3 generated by the full twist obtained by twisting the cylinder keeping the bottom fixed and turning the top by the angle 2π . Respective facts hold for loops in $C_3(\mathbb{C})$ with specified boundary values instead of loops with a base point. With the natural definition of the extremal length with totally real boundary values of a pure 3-braid b this extremal length is equal to $\lambda_{tr}(\mathfrak{C}_*(b))$. The respective fact holds for perpendicular bisector boundary values. The obtained invariants are invariants of 3-braids rather than invariants of conjugacy classes of 3-braids. In particular, they are finer than a popular invariant of braids, the entropy. Our estimates imply estimates of the entropy of pure 3-braids b in terms of the representing word of the image $\mathfrak{C}_*(b)$. Notice that the name "totally real" and "perpendicular bisector" is motivated by the definition in the case of braids. Details will be given in a later paper. For an introduction to braids see e.g. [2]. For more information on the conformal module, the extremal length and entropy of braids, or of conjugacy classes of braids, respectively, see also [3] and [4].

We will now lift the elements of π_1^{pb} to the logarithmic covering U_{\log} of $\mathbb{C} \setminus \{-1, 1\}$ and identify the lifts with homotopy slalom curves. This geometric interpretation will suggest how to estimate the extremal length with perpendicular bisector boundary values.

The logarithmic covering of $\mathbb{C} \setminus \{-1, 1\}$ is the universal covering of the twice punctured Riemann sphere $\mathbb{P}^1 \setminus \{-1, 1\}$ with all preimages of ∞ under the covering map removed. Geometrically the universal covering of $\mathbb{P}^1 \setminus \{-1, 1\}$ can be described as follows. Take copies of $\mathbb{P}^1 \setminus (-1, 1)$ labeled by the set \mathbb{Z} of integer numbers. Close up each copy by attaching two copies of $(-1, 1)$, the $+$ -edge (the accumulation set of points of the upper half-plane) and the $-$ -edge (the accumulation set of points of the lower half-plane). For each $k \in \mathbb{Z}$ we glue the $+$ -edge of the k -th copy to the $-$ -edge of the $k+1$ -st copy (using the identity mapping on $(-1, 1)$ to identify points on different edges). Denote by U_{\log} the set obtained from the described covering by removing all preimages of ∞ .

The following proposition holds.

Proposition 1. *The set U_{\log} is conformally equivalent to $\mathbb{C} \setminus i\mathbb{Z}$. The mapping $f_1 \circ f_2$, $f_2(z) = \frac{e^{\pi z} - 1}{e^{\pi z} + 1}$, $z \in \mathbb{C} \setminus i\mathbb{Z}$, $f_1(w) = \frac{1}{2}(w + \frac{1}{w})$, $w \in \mathbb{C} \setminus \{0\}$, is a covering map from $\mathbb{C} \setminus i\mathbb{Z}$ to $\mathbb{C} \setminus \{-1, 1\}$.*

The lift of α_1 with initial point $\frac{-i}{2} + ik$ is a curve which joins $\frac{-i}{2} + ik$ with $\frac{-i}{2} + i(k+1)$ and is contained in the closed left half-plane. The only points on the imaginary axis are the endpoints.

The lift of α_2 with initial point $\frac{-i}{2} + ik$ is a curve which joins $\frac{-i}{2} + ik$ with $\frac{-i}{2} + i(k-1)$ and is contained in the closed right half-plane. The only points on the imaginary axis are the endpoints.

Figure 1 shows the curves α_1 and α_2 which represent the generators of the fundamental group $\pi_1(\mathbb{C} \setminus \{-1, 1\}, 0)$ and their lifts under the covering maps f_1 and $f_2 \circ f_1$. For $j = 1, 2$ the curves α'_j and α''_j are the two lifts of α_j under the double branched covering $f_1 : \mathbb{C} \setminus \{0\} \rightarrow \mathbb{C}$ with branch points 1 and -1 . The curve $\tilde{\alpha}'_1$ is the lift of α'_1 under the mapping f_2 with initial point $\frac{-i}{2}$, the curve $\tilde{\alpha}'_2$ lifts α'_2 and has initial point $\frac{i}{2}$.

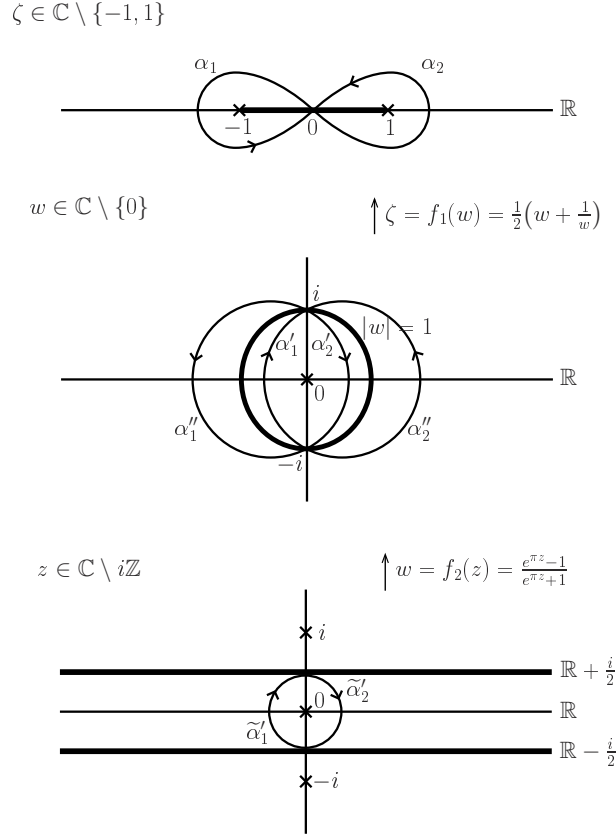


Figure 1

Consider the curve α_1^n , $n \in \mathbb{Z} \setminus \{0\}$. It runs n times along the curve α_1 if $n > 0$ and $|n|$ times along the curve which is inverse to α_1 if $n < 0$. For each $k \in \mathbb{Z}$ the curve α_1^n lifts to a curve with initial point $\frac{-i}{2} + ik$ and terminating point $\frac{-i}{2} + ik + in$ which is contained in the closed left half-plane and omits the points in $i\mathbb{Z}$. Respectively, α_2^n , $n \in \mathbb{Z} \setminus \{0\}$, lifts to a curve with initial point $\frac{i}{2} + ik$ and terminating point $\frac{i}{2} + ik - in$ which is contained in the closed right half-plane and omits the points in $i\mathbb{Z}$. The mentioned lifts are homotopic through curves in $\mathbb{C} \setminus i\mathbb{Z}$ with

endpoints on $i\mathbb{R} \setminus i\mathbb{Z}$ to curves with interior contained in the open (left, respectively, right) half-plane. We have the following definition where we identify a curve with its image, ignoring orientation.

Definition 2. A simple arc in $\mathbb{C} \setminus i\mathbb{Z}$ with endpoints on different connected components of $i\mathbb{R} \setminus i\mathbb{Z}$ is called an elementary slalom curve if its interior (i.e. the complement of its endpoints) is contained in one of the open half-planes $\mathbb{C}_r \stackrel{\text{def}}{=} \{z \in \mathbb{C} : \operatorname{Re} z > 0\}$ or $\mathbb{C}_\ell \stackrel{\text{def}}{=} \{z \in \mathbb{C} : \operatorname{Re} z < 0\}$.

A curve in $\mathbb{C} \setminus i\mathbb{Z}$ is called an elementary half slalom curve if one of the endpoints is contained in a horizontal line $\{\operatorname{Im} z = k + \frac{1}{2}\}$ for an integer k and the union of the curve with its mirror reflection in the line $\{\operatorname{Im} z = k + \frac{1}{2}\}$ is an elementary slalom curve.

A slalom curve in $\mathbb{C} \setminus i\mathbb{Z}$ is a curve which can be divided into a finite number of elementary slalom curves so that consecutive elementary slalom curves are contained in different half-planes.

A curve which is homotopic to a slalom curve in $\mathbb{C} \setminus i\mathbb{Z}$ through curves with endpoints in $\mathbb{R} \setminus i\mathbb{Z}$ is called a homotopy slalom curve.

Figure 2 below shows a slalom curve which represents a lift of the element $a_2^{-1} a_1^2 a_2^{-3} a_1^{-1} a_2^{-1} a_1^{-1} a_2 a_1^{-1}$ with perpendicular bisector boundary values.

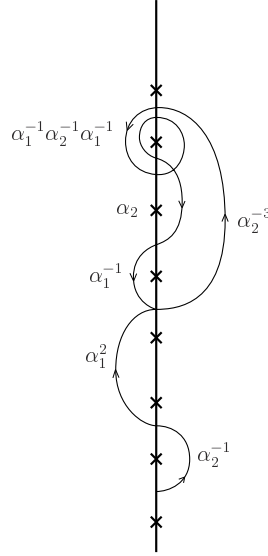


Figure 2

Elementary slalom curves and elementary half slalom curves will serve as building blocks. Note that each curve in $\mathbb{C} \setminus i\mathbb{Z}$ with endpoints in $i\mathbb{R} \setminus i\mathbb{Z}$ is a homotopy slalom curve or is homotopic to the identity in $\mathbb{C} \setminus i\mathbb{Z}$ with endpoints in $i\mathbb{R} \setminus i\mathbb{Z}$. Proposition 1 implies that each

lift of a curve in $\mathbb{C} \setminus \{-1, 1\}$ with endpoints on the imaginary axis is of such type. The extremal length of slalom curves (more precisely of homotopy classes of slalom curves) can be defined in the same way as the respective object for elements of the fundamental group π_1^{pb} . The extremal length with perpendicular bisector boundary values of an element of π_1 is equal to that of its lift.

Consider the extremal length of an elementary slalom curve which corresponds to the word $a^n \in \pi_1$ where a equals either a_1 or a_2 . Without loss of generality we may assume that $a = a_1$, hence the curve is contained in the closed left half-plane. After a translation the endpoints of the curve are contained in the intervals $(-i(M+1), -iM)$ and $(iM, i(M+1))$, respectively, with $M = \frac{|n|-1}{2}$. For $M = 0$ (thus for $|n| = 1$) the extremal length equals 0. In this case we call the original curve a trivial elementary slalom curve. Let M be positive. The curve is represented by the extension to the boundary of a conformal mapping of an open rectangle \mathcal{R}^M onto the left half-plane which maps the horizontal sides onto $[-i(M+1), -iM]$, and $[iM, i(M+1)]$, respectively. Hence its extremal length is bounded from above by the extremal length of the rectangle \mathcal{R}^M .

The conformal mapping of a rectangle onto the left half-plane \mathbb{C}_ℓ whose extension to the boundary maps the horizontal sides onto $[-i(M+1), -iM]$, and $[iM, i(M+1)]$, respectively, is related to elliptic integrals. With a suitable normalization of the rectangle the inverse of the mapping is equal to the elliptic integral

$$\begin{aligned} \mathcal{F}_M(z) &= \int_0^z \frac{d\zeta}{\sqrt{(\zeta^2 - (iM)^2)(\zeta^2 - (i(M+1))^2)}} \\ &= \frac{i}{M} \int_0^{\frac{z}{iM}} \frac{dw}{\sqrt{(1-w^2)((1+\frac{1}{M})^2 - w^2)}}, \quad z \in \mathbb{C}_\ell. \end{aligned} \quad (1)$$

We use the branch of the square root which is positive on the positive real axis. The function \mathcal{F}_M extends continuously to the imaginary axis (the integral converges). The extended map maps the closed left half-plane to a closed rectangle. The points $-i(M+1)$, $-iM$, iM and $i(M+1)$ are mapped to the vertices of the rectangle. It is known and follows from formula (1) for the elliptic integral that for $M \geq \frac{1}{2}$ the extremal length of the rectangle \mathcal{R}^M satisfies the inequalities

$$c \log(1+M) \leq \lambda(\mathcal{R}^M) \leq C \log(M+1) \quad (2)$$

for positive constants c and C not depending on M .

Equation (2) suggests the following proposition.

Proposition 2. *The extremal length $\lambda_{k,\ell}$ of an elementary slalom curve with endpoints in the intervals $(ik, i(k+1))$ and $(i\ell, i(\ell+1))$, respectively, with $|k - \ell| \geq 2$, satisfies the inequalities*

$$c' \log\left(1 + \frac{|k - \ell| - 1}{2}\right) \leq \lambda_{k,\ell} \leq C' \log\left(1 + \frac{|k - \ell| - 1}{2}\right) \quad (3)$$

for positive constants c' and C' not depending on k and ℓ .

There are explicit estimates for the constants c' and C' .

The proof will be given elsewhere. Here we already discussed the estimate from above. The estimate from below is more subtle. The first difficulty is that the representing mappings for an elementary slalom curve are not necessarily conformal mappings, they are merely holomorphic. The second difficulty is that the image of the rectangle is not necessarily contained in the half plane. We can only say about the mapping that it lifts to a holomorphic mapping into the universal covering of $\mathbb{C} \setminus i\mathbb{Z}$ with specified boundary values. The universal covering is a half-plane, but the horizontal sides of the rectangle are not mapped any more into boundary intervals of the half-plane but into some curves in the half-plane. One tool for dealing with these difficulties is an analog of the following lemma which is of independent interest.

Lemma 1. *Let \mathcal{R}_1 and \mathcal{R}_2 be rectangles with sides parallel to the axes. Suppose S_2 is a vertical strip bounded by the two vertical lines which are prolongations of the vertical sides of the rectangle \mathcal{R}_2 . Let $f : \mathcal{R}_1 \rightarrow S_2$ be a holomorphic map whose extension to the closure maps the two horizontal sides of \mathcal{R}_1 into different horizontal sides of \mathcal{R}_2 . Then*

$$\lambda(\mathcal{R}_1) \geq \lambda(\mathcal{R}_2).$$

Equality holds if and only if the mapping is a surjective conformal map from \mathcal{R}_1 to \mathcal{R}_2 .

The proof of the lemma is based on the Cauchy-Riemann equations.

To estimate the extremal length of an arbitrary element of the fundamental group $\pi_1 = \pi_1(\mathbb{C} \setminus \{-1, 1\}, \{0\})$ we represent the element as a word in the generators (and identify it with the word). A word is in reduced form (or a reduced word) if it is written as product of powers of generators where consecutive terms correspond to different generators. Consider the reduced word

$$w = a_1^{n_1} \cdot a_2^{n_2} \cdot \dots, \quad (4)$$

where the n_j are integers. (Here $a_j^0 \stackrel{\text{def}}{=} \text{id}$, we allow $n_1 = 0$.) We are interested first in the extremal length with perpendicular boundary value conditions. One can show that any curve which represents this element with perpendicular boundary values can be represented as composition of curves $\alpha_1^{n_1}, \alpha_2^{n_2}, \dots$, which represent $a_1^{n_1}, a_2^{n_2}, \dots$, with perpendicular

boundary values. Together with Theorems 2 and 4 of [1] this implies the following estimate from below

$$\lambda_{pb}(a_1^{n_1} a_2^{n_2} \dots) \geq \lambda_{pb}(a_1^{n_1}) + \lambda_{pb}(a_2^{n_2}) + \dots$$

This gives a good lower bound if all terms $a_j^{n_j}$ of the reduced word enter with power of absolute value at least 2. It does not give a good lower bound, for example, for the word $(a_1 a_2^{-1})^n$ with $n \geq 1$, or for the word $a_1^{n_1} (a_2 a_1)^{n_2}$ for integers n_1 and n_2 larger than 1 and n_2 much bigger than n_1 . In the first example the reason is the following. Each representing curve for $a_1 a_2^{-1}$ with pb -boundary values can be written as composition of the following two curves: a curve α_1 with pb -boundary values on the left and tr -boundary values on the right representing a_1 , and a curve α_2^{-1} with tr -boundary values on the left and pb -boundary values on the right representing a_2^{-1} . The lift of each of the two curves is a non-trivial half-slalom curve. Hence the extremal length with pb -boundary values of the element $(a_1 a_2^{-1})^n$ is proportional to n .

For the second example one can show that each representing curve with pb -boundary values contains a piece corresponding to $(a_2 a_1)^{n_2}$ with mixed boundary values. A different choice of a lift gives a half slalom curve which shows that the extremal length of this piece is proportional to $\log(\frac{n_2-1}{2})$.

The discussion suggests that the extremal length of a general element of π_1^{pb} can be given in terms of a syllable decomposition of the representing reduced word.

We describe now the syllable decomposition of the word (4).

- (1) Any term $a_j^{n_j}$ of the reduced word with $|n_j| \geq 2$ is a syllable.
- (2) Any maximal sequence of consecutive terms of the reduced word which have equal power equal to either $+1$ or -1 is a syllable.
- (3) Each remaining term of the reduced word is characterized by the following properties. It enters with power $+1$ or -1 and the neighbouring term on the right (if there is one) and also the neighbouring term on the left (if there is one) has power different from that of the given one. Each term of this type is a syllable, called a singleton.

Define the degree of a syllable $\deg(\text{syllable})$ to be the sum of the absolute values of the powers of terms entering the syllable.

For example, the syllables of the word $a_2^{-1} a_1^2 a_2^{-3} a_1^{-1} a_2^{-1} a_1^{-1} a_2 a_1^{-1}$ (see Figure 2) from left to right are the singleton a_2^{-1} , the syllable a_1^2 of degree 2, the syllable a_2^{-3} of degree 3, the syllable $a_1^{-1} a_2^{-1} a_1^{-1}$ of degree 3, the singleton a_2 and the singleton a_1^{-1} .

Put $\Lambda(w) \stackrel{\text{def}}{=} \sum_{\text{syllables of } w} \log(1 + \deg(\text{syllable}))$.

The following theorem holds.

Theorem 1. *There are absolute positive constants C_+ and C_- such that the following holds. Let w be the word representing an element of $\pi_1 = \pi_1(\mathbb{C} \setminus \{-1, 1\}, \{0\})$. Then*

- (1) $C_- \cdot \Lambda(w) \leq \lambda_{tr}(w) \leq C_+ \cdot \Lambda(w)$, *except in the following cases: $w = a_1^n$ or $w = a_2^n$ for an integer n . In these cases $\lambda_{tr}(w) = 0$.*
- (2) $C_- \cdot \Lambda(w) \leq \lambda_{pb}(w) \leq C_+ \cdot \Lambda(w)$, *except in the following case: each term in the reduced word w has the same power, which equals either $+1$ or -1 . In these cases $\lambda_{pb}(w) = 0$.*

Corollary 1. *For an element $w \in \pi_1$ which is not one of the exceptional cases of Theorem 1 the two versions of the extremal length are comparable:*

$$C_1 \lambda_{tr}(w) \leq \lambda_{pb}(w) \leq C_2 \lambda_{tr}(w)$$

for positive constants C_1 and C_2 which do not depend on w .

Corollary 2. *There are positive constants C'_- and C'_+ such that for each element $w \in \pi_1$ which is not a singleton the estimate*

$$C'_- \cdot \Lambda(w) \leq \lambda_{tr}(w) + \lambda_{pb}(w) \leq C'_+ \cdot \Lambda(w)$$

holds.

The extremal length of elements of the fundamental group of the complex plane with an arbitrary number of punctures will be treated in a forthcoming paper. The case of n -braids with arbitrary n is more subtle.

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